Let us discuss about escape speed and orbital speed,satellites ,orbital speed and time period,energy of an orbiting satellite,geo-stationary and polar satellite and weightlessness .

Expression for escape speed:

- Consider an object of mass M on the surface of the Earth.
- When it is thrownup with an initial speed v_i, the initial total energy of the object is

$$E_i = \frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} \qquad (6.53)$$

- Where M_E is the mass of earth and R_E radius of the earth.
- The term is the potential energy of the mass M.
- When the object reaches a height far away from the earth and hence treated as approaching infinity,the gravitational potential energy becomes zero as well
- Therefore the final total energy of the object becomes zero.
- This is for minimum energy and for minimum speed to escape.otherwise kinetic energy can be non-zero.

$$E_f = 0$$

According to the law of energy conservation,

$$E_i = E_f \qquad (6.54)$$

Substituting (6.53) in (6.54) we get,

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \qquad (6.55)$$

- The escape speed ,the minimum speed required by an object to escape from earths gravitational field,replacing v_i with v_e

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E}$$

$$v_e^2 = \frac{GMM_E}{R_E}.\frac{2}{M}$$

$$v_e^2 = \frac{2GM_E}{R_E}$$

Using $g = \frac{GM_E}{R_E^2}$,

$$v_e^2 = 2gR_E$$

$$v_e = \sqrt{2gR_E} \qquad (6.56)$$

- From equation 6.56 the escape speed depends on two factors.acceleration due to gravity and radius of the earth.
- By substituting the values of g (9.8m/s^2) and R_E=6400 KM,the escape speed of earth is v_e=11.2km/s.

Expression for time period of satellite orbiting the earth:

The distance covered by the satellite during one rotation in its orbit is equal to $2\pi(R_E + h)$ and time taken for it is the time period, T. Then

$$Speed\ v = \frac{Distance\ travelled}{Time\ taken} = \frac{2\pi(R_E + h)}{T}$$

$$\sqrt{\frac{GM_E}{(R_E+h)}} = \frac{2\pi(R_E+h)}{T} \quad (6.59)$$

$$T = \frac{2\pi}{\sqrt{GM_E}}(R_E+h)^{3/2} \quad (6.60)$$

Squaring both sides of the equation (6.60), we get

$$T^2 = \frac{4\pi^2}{GM_E}(R_E+h)^3$$

$$\frac{4\pi^2}{GM_E} = \text{constant say } c$$

$$T^2 = c(R_E+h)^3 \quad (6.61)$$

Equation (6.61) implies that a satellite orbiting the Earth has the same relation between time and distance as that of Kepler's law of planetary motion. For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E. Then,

$$T^2 = \frac{4\pi^2}{GM_E} R_E^{\ 3}$$

$$T^2 = \frac{4\pi^2}{GM_E / R_E^{\ 2}} R_E$$

$$T^2 = \frac{4\pi^2}{g} R_E$$

$$since \ GM_E / R_E^{\ 2} = g$$

$$T = 2\pi\sqrt{\frac{R_E}{g}} \quad (6.62)$$

By substituting the values of $R_E = 6.4 \times 10^6$ m and g = 9.8 m s^{-2}, the orbital time period is obtained as T ≃ 85 minutes.

Energy of an orbiting satellite:

The total energy of a satellite orbiting the earth at a distance h from the surface of the earth.

- The energy of the satellite is sum of its kinetic energy and gravitational potential energy
- The potential energy of the satellite is

$$U = -\frac{GM_sM_E}{(R_E + h)} \qquad (6.63)$$

Here M_s - mass of the satellite, M_E - mass of the Earth, R_E - radius of the Earth.

The Kinetic energy of the satellite is

$$K.E = \frac{1}{2}M_sv^2 \qquad (6.64)$$

Here v is the orbital speed of the satellite and is equal to

$$v = \sqrt{\frac{GM_E}{(R_E + h)}} \qquad (6.65)$$

Substituting the value of v in (6.64), the kinetic energy of the satellite becomes,

$$K.E = \frac{1}{2}\frac{GM_EM_s}{(R_E + h)}$$

Weightlessness:

- Objects on earth experience the gravitational force on earth.
- The gravitational force acting on an object of mass m is mg.
- The weight w is defined as downward force whose magnitude w is equal to that of upward forc that must be applied to object to it rest or at constant velocity relative to the earth.
- The direction of weight is in the direction of gravitational force.
- The magnitude of weight of an object is denoted as w=N=mg'

Apparent weight in elevators.:

Let us consider a man inside an elevator in the following scenarios.

- When a man is standing in the elevator there are two factors acting no him.
- Gravitational force which acts downward .if we take the vertical direction as positive y-direction the gravitational force acting on the man is

$$\vec{F}_G = -mg\hat{j}$$

- The normal force exerted by floor on the man which acts vertically upward,

$$\vec{N} = N\hat{j}$$

- **Case1**:when the elevator is at rest:

The acceleration of the man is zero. Therefore the net force acting on the man is zero. With respect to inertial frame (ground), applying Newton's second law on the man,

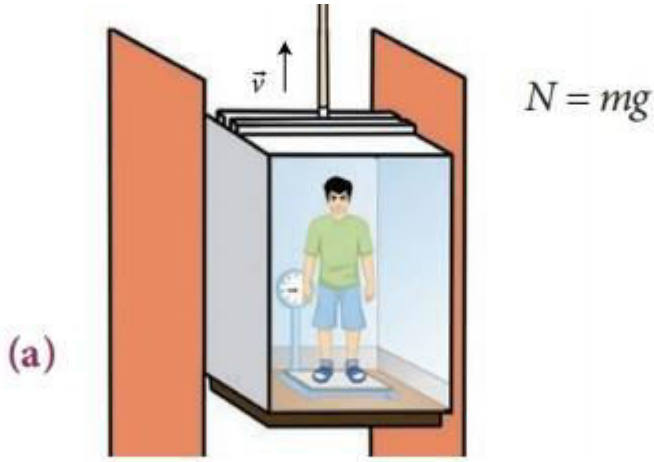
$$\vec{F}_G + \vec{N} = 0$$
$$-mg\hat{j} + N\hat{j} = 0$$

By comparing the components, we can write

$$N - mg = 0 \text{ (or) } N = mg \qquad (6.67)$$

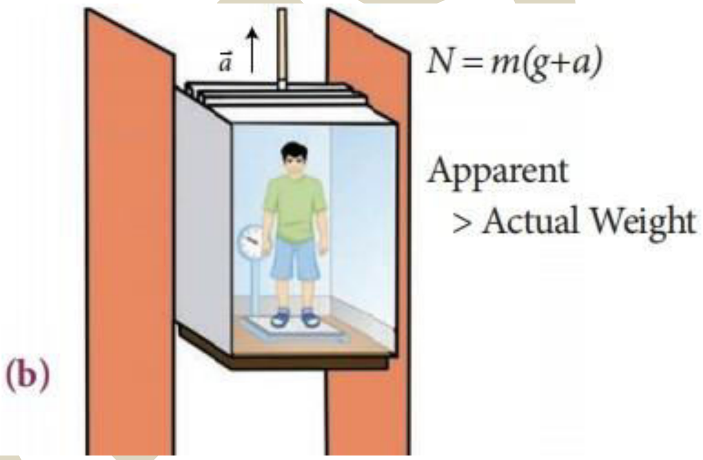
Since weight, W =N, the apparent weight of the man is equal to his actual weight.

- **Case2**:when elevator is moving uniformly in the upward or downward direction:



- In the uniform motion (constant velocity)thr net force acting on the man is still zero.
- Hence ,in this case also the aaparent isweight on the man is equl to his actual weight as shown in figure (a).

Case 3:when the elevator is accelerating upwards:



If an elevator is moving with upward acceleration ($\vec{a} = a\hat{j}$) with respect to inertial frame (ground), applying Newton's second law on the man,

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

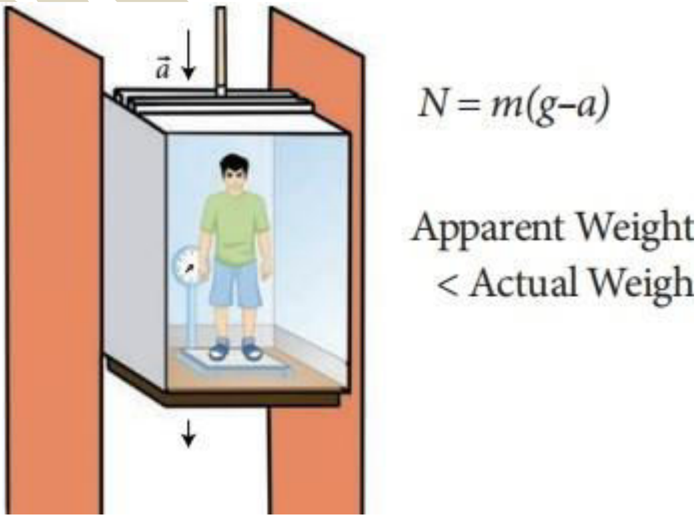
$$-mg\hat{j} + N\hat{j} = ma\hat{j}$$

By comparing the components,

$$N = m(g + a) \qquad (6.68)$$

Therefore, apparent weight of the man is greater than his actual weight. It is shown in Figure 6.23(b)

❖ **Case 4:** when the elevator is accelerating downwards:



(c)

If the elevator is moving with downward acceleration ($\vec{a} = -a\hat{j}$), by applying Newton's second law on the man, we can write

$$\vec{F}_G + \vec{N} = m\vec{a}$$

Writing the above equation in terms of unit vector in the vertical direction,

$$-mg\hat{j} + N\hat{j} = -ma\hat{j}$$

By comparing the components,

$$N = m(g - a) \qquad (6.69)$$

Therefore, apparent weight W = N = m(g-a) of the man is lesser than his actual weight. It is shown in Figure 6.23(c)

❖ Weightlessness of freely bodies.;

Free Fall
N = 0; g = a

(d)

- Freely falling objects experience only gravitational force.
- As they are freely falling ,they are not in contact with any surface(by neglecting air friction).
- The normal force acting on the object is zero.
- The downward force is equal to the acceleration due to gravity of the earth I.e.a=g.

From equation 6.69,we get.

$$a = g \quad \therefore N = m(g - g) = 0.$$

This is called the state of weightlessness. When the lift falls (when the lift wire cuts) with downward acceleration a = g, the person inside the elevator is in the state of weightlessness or free fall. It is shown in Figure 6.23(d)

Geo -stationary and polar satellite:

- The satellites orbiting the Earth havedifferent time periods corresponding to different orbital radii.

Kepler's third law is used to find the radius of the orbit.

Geostationary satellite and polar satellite:

- The satellites orbiting the Earth havedifferent time periods corresponding to different orbital radii.

Kepler's third law is used to find the radius of the orbit.

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$$

$$(R_E + h)^3 = \frac{GM_E T^2}{4\pi^2}$$

$$R_E + h = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$$

Substituting for the time period (24 hrs = 86400 seconds), mass, and radius of the Earth, *h* turns out to be 36,000 km. Such satellites are called "geo-stationary satellites", since they appear to be stationary when seen from Earth.

- India uses the INSAT group of satellites that are basically geo-stationary satellites for the purpose of telecommunication. Another type of satellite which is placed at a distance of 500 to 800 km from the surface of the earth orbits the earth from north to south direction.
- The time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day.
- A Polar satellite covers a small strip of areafrom pole to pole during one revolution. In the next revolution it covers a different stripof area since the Earth would have moved by a small angle. In this way polar satellitescover the entire surface area of the Earth.

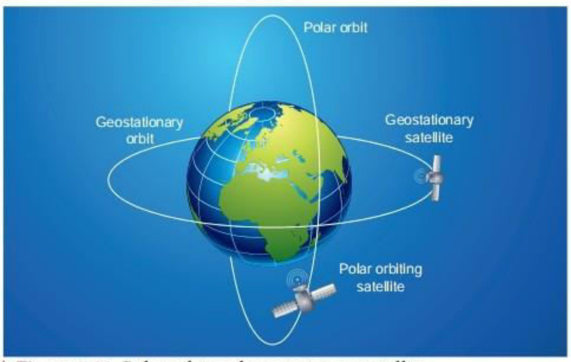


Figure 6.21 Polar orbit and geostationary satellite

Heliocentric system over geocentric system:

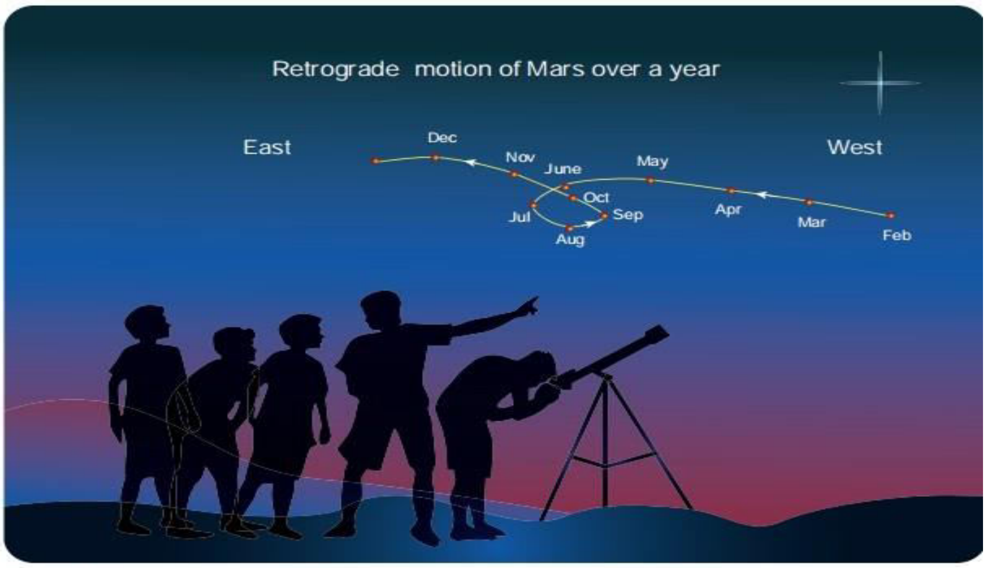


Figure 6.25 Retrograde motion of planets

- it can be seen that the planets move eastwards and reverse their motion for a while and return to eastward motion again. This is called "retrograde motion" of planets.figure.
- To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model.
- According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle".
- According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle".
- A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth
- But Ptolemy's model became more and more complex as every planet was foundto undergo retrograde motion
- the Polish astronomer Copernicus proposed the heliocentric model to explain this problem in a simpler manner. Accordingto this model, the Sun is at the center ofthe solar system and all planets orbited the Sun.

. The retrograde motion of planets withrespect to Earth is because of the relative motion of the planet with respect to Earth.

Measurement o radius of the earth:

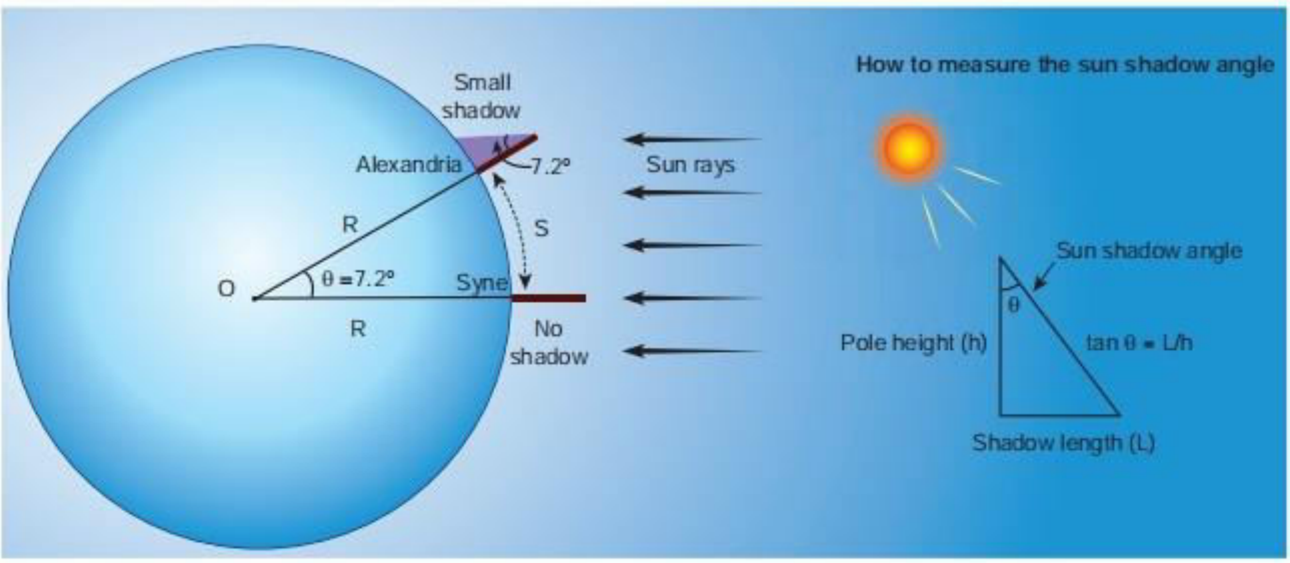


Figure 6.30 Measuring radius of The Earth

- during noon time of summer solstice the Sun's rayscast no shadow in the city Syne which was located 500 miles away from Alexandria

- At the same day and same time he found that in Alexandria the Sun's rays made 7.2 degree with local vertical as shown in the Figure 6.30

- He realized that this differenceof 7.2 degree was due to the curvature of theEarth.

The angle 7.2 degree is equivalent to $\frac{1}{8}$ radian. So $\theta = \frac{1}{8}$ rad;

If S is the length of the arc between the cities of Syne and Alexandria, and if R is radius of Earth, then

$$S = R\theta = 500 \text{ miles},$$

so radius of the Earth

$$R = \frac{500}{\theta} \text{ miles}$$

$$R = \frac{500}{\left(\frac{1}{8}\right)} \text{miles}$$

$$R = 4000 \text{ miles}$$

1 mile is equal to 1.609 km. So, he measured the radius of the Earth to be equal to R = 6436 km, which is amazingly close to the correct value of 6378 km.

Measurement of earth shadow(umbra) radius during total lunar eclipse:

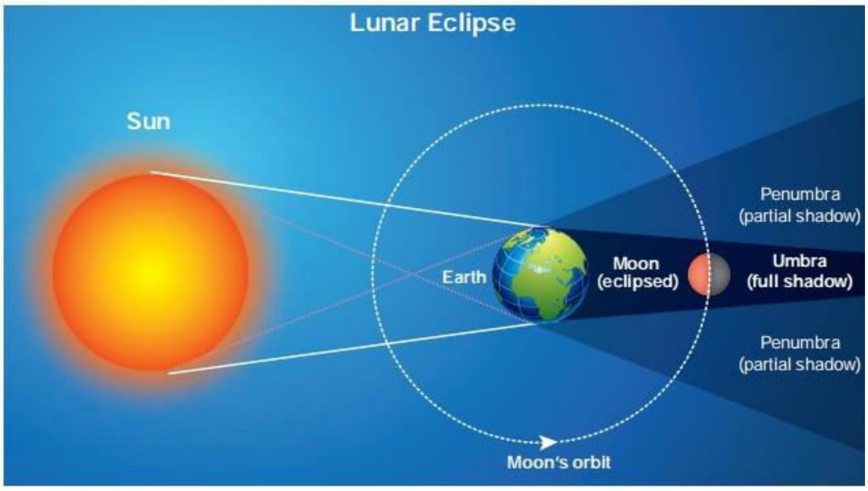


Figure 6.31 Total lunar eclipse

- It ispossible to measure the radius of shadowof the Earth at the point where the Mooncrosses. Figure 6.31 illustrates this.
- When the Moon is inside the umbra shadow, it appears red in color. As soon as the Moon exits from the umbra shadow ,it appears in crescent shape.
- By finding the apparent radii of the earths' umbra shadow and moon ,the ratio of these radii can be calculated.
- The apparent radius of earths' umbra shadow $=R_S=13.3$ cm.

The apparent radius of the Moon $= R_m = 5.15$ cm

The ratio $\frac{R_s}{R_m} \approx 2.56$

The radius of the Earth's umbra shadow is $R_s = 2.56 \times R_m$

The radius of Moon $R_m = 1737$ km

The radius of the Earth's umbra shadow is $R_s = 2.56 \times 1737\,\text{km} \cong 4446\,\text{km}$.

The correct radius is 4610 km.

The percentage of error in the calculation

$$= \frac{4610 - 4446}{4610} \times 100 = 3.5\%.$$

Why there is no lunar eclipse and solar eclipse every month?

- Moons orbit is tilted 5 with respect to earths' orbit.
- Due to this 5 tilt only certain periods of the year ,the sun,earth,and moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment.