Greetings ! Dear students in the previous notes we learn about the concept of Functions and their example problems, In this notes we will discuss Exercise 1.3 problems.

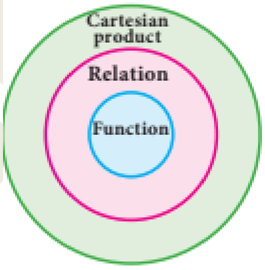
Function

* A function is defined as a relation between a set of inputs having one output each.
* In simple words, a function is a relationship between inputs where each input is related to exactly one output.
* Every function has a domain and co domain or range.
* A function is generally denoted by f(x) where x is the input.
* The general representation of a function is y = f(x).
* Among several relations that exist between two non-empty sets, some special relations are important for further exploration. Such relations are called **"Functions".**

Definition : From text book

A relation f between two non-empty sets X and Y is called a **function** from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$.

That is, $f = \{(x,y) \mid \text{for all } x \in X, y \in Y\}$.



A function f can be thought as a mechanism (or device) (Fig.1.12(b)), which gives a unique output f(x) to every input x.

Now we are enter into exercise problems.

1) Let $f = \{(x, y) \mid x, y \in N \text{ and } y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function?

Solution:

$F = \{(x, y) | x, y \in N \text{ and } y = 2x\}$

$x = \{1, 2, 3, \ldots\}$

$y = \{1 \times 2, 2 \times 2, 3 \times 2, 4 \times 2, 5 \times 2 \ldots\}$

$R = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), \ldots\}$

Domain of R = $\{1, 2, 3, 4, \ldots\}$,

Co-domain = $\{1, 2, 3 \ldots\}$

Range of R = $\{2, 4, 6, 8, 10, \ldots\}$

Yes, this relation is a function.

2) Let X = {3, 4, 6, 8}. Determine whether the relation $R = \{(x, (f x)) \mid, x \in X\ f(x) = x^2 + 1\}$ is a function from X to N?

Solution:

$x = \{3, 4, 6, 8\}$

$R = ((x, f(x)) | x \in X, f(x) = X^2 + 1\}$

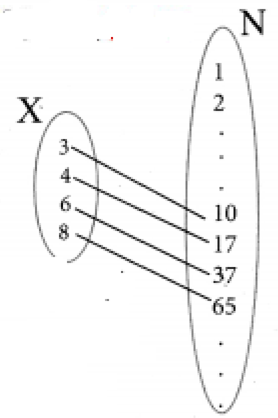
$f(x) = x^2 + 1$

$f(3) = 3^2 + 1 = 10$

$f(4) = 4^2 + 1 = 17$

$f(6) = 6^2 + 1 = 37$

$f(8) = 8^2 + 1 = 65$



$R = \{(3, 10), (4, 17), (6, 37), (8, 65)\}$

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Yes, R is a function from X to N.

3) Given the function f : $x^2 - 5x + 6$, evaluate (i) f (-1) (ii) f (2a) (iii) f (2) (iv) f (x-1)

Answer:

$f(x) = x^2 - 5x + 6$

(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

(iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

(iv) $f(x - 1) = (x - 1)^2 - 5(x - 1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6$

$= x^2 - 7x + 12$

4) A graph representing the function $f(x)$ is given in Fig.1.16 it is clear that $f(9) = 2$.

(i) Find the following values of the function

(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$

(ii) For what value of x is $f(x) = 1$?

(iii) Describe the following (i) Domain (ii) Range.

(iv) What is the image of 6 under f?

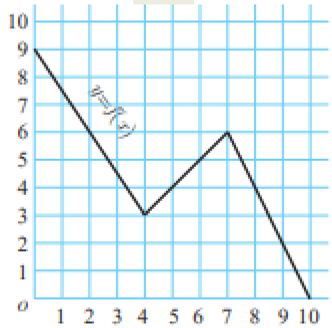


Fig. 1.16

Solution:

From the graph

(a) f(0) = 9

(b) f(7) = 6

(c) f(2) = 6

(d) f(10) = 0

(ii) At x = 9.5, f(x) = 1

(iii) Domain = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$= \{x \mid 0 < x < 10, x \in R\}$

Range $= \{x \mid 0 < x < 9, x \in R\}$

= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

(iv) The image of 6 under f is 5.

5) Let $f(x) = 2x+5$. If $x \neq 0$ then find $\frac{f(x+2)-f(2)}{x}$.

Solution:

Given f(x) = 2x + 5, x ≠ 0.

$\frac{f(x+2)-f(2)}{x}$

$$\begin{aligned} f(x) &= 2x+5 \\ &\Rightarrow f(x+2) \\ &= 2(x+2)+5 \\ &= 2x+4+5 = 2x+9 \\ \Rightarrow f(2) &= 2(2)+5 = 4+5 = 9 \end{aligned}$$

$$\therefore \frac{f(x+2)-f(2)}{x} = \frac{2x+9-9}{x} = \frac{2x}{x} = 2$$

6) A function f is defined by $f(x) = 2x - 3$

(i) find $\frac{f(0)+f(1)}{2}$.

(ii) find x such that $f(x) = 0$.

(iii) find x such that $f(x) = x$.

(iv) find x such that $f(x) = f(1-x)$.

Solution:

Given f(x) = 2x – 3

(i) find $\frac{f(0)+f(1)}{2}$

f(0) = 2(0) – 3 = -3

f(1) = 2(1) – 3 = -1

$\therefore \frac{f(0)+f(1)}{2} = \frac{-3-1}{2} = \frac{-4}{2} = -2$

(ii) f(x) = 0

⇒ 2x – 3 = 0

2x = 3

x = $\frac{3}{2}$

(iii) f(x) = x

⇒ 2x – 3 = x ⇒ 2x – x = 3

x = 3

(iv) $f(x) = f(1 - x)$

$2x - 3 = 2(1 - x) - 3$

$2x - 3 = 2x - 2x - 3$

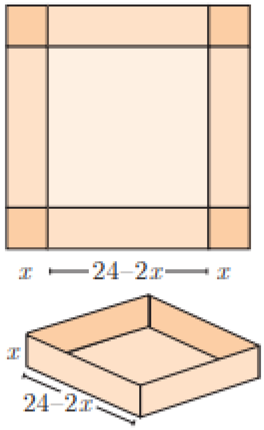
$2x + 2x = 2 - 3 + 3$

$4x = 2$

$x = \frac{2}{4}$

$x = \frac{1}{2}$

7) An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown (Fig.1.17). Express the volume V of the box as a function of x.



Solution:

Volume of the box = Volume of the cuboid

= l × b × h cu. units

Here l = 24 – 2x

b = 24 – 2x

h = x

$\therefore V = (24 - 2x)(24 - 2x) \times x$

$= (576 - 48x - 48x + 4x^2)x$

$V = 4x^3 - 96x^2 + 576x$

8) A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$

Solution:

$f(x) = 3 - 2x$

$f(x^2) = 3 - 2x^2$

$$\begin{aligned}(f(x))^2 &= (3-2x)^2 = 9 - 12x + 4x^2 \\ f(x^2) &= (f(x))^2 \Rightarrow 3 - 2x^2 = 9 - 12x + 4x^2 \\ 6x^2 - 12x + 6 &= 0\ [\div 6] \\ x^2 - 2x + 1 &= 0 \\ (x-1)(x-1) &= 0 \\ x &= 1, 1\end{aligned}$$

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9) A plane is flying at a speed of 500 km per hour. Express the distance 'd ' travelled by the plane as function of time t in hours.

Answer:

Speed of the plane = 500 km/hr

Distance travelled in "t" hours

= 500 × t (distance = speed × time)

= 500 t

10) The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length(x) as $y = ax + b$, where a, b are constants.

Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.

(ii) Find a and b.

(iii) Find the height of a person whose forehand length is 40 cm.

(iv) Find the length of forehand of a person if the height is 53.3 inches.

Solution:

(i) Given y = ax + b .... (1)

The ordered pairs are R = {(35, 56) (45, 65) (50, 69.5) (55, 74)}

∴ Hence this relation is a function.

(ii) Consider any two ordered pairs $(\overset{x}{35}, \overset{y}{56})$

$(\overset{x}{45}, \overset{y}{65})$ substituting in (1) we get,

$$65 = 45a + b \quad ....(2)$$
$$56 = 35a + b \quad ....(3)$$

Subtracting, $9 = 10a$

$$\therefore a = \frac{9}{10} = 0.9$$

Substituting a = 0.9 in (2) we get

$\Rightarrow 65 = 45(.9) + b$

$\Rightarrow 65 = 40.5 + b$

$\Rightarrow b = 65 - 40.5$

$\Rightarrow b = 24.5$

$\therefore a = 0.9, b = 24.5$

$\therefore y = 0.9x + 24.5$

(iii) Given x = 40 , y = ?

$\therefore (4) \rightarrow y = 0.9\ (40) + 24.5$

$\Rightarrow y = 36 + 24.5$

$\Rightarrow y = 60.5$ inches

(iv) Given y = 53.3 inches, x = ?

$(4) \rightarrow 53.3 = 0.9x + 24.5$

$\Rightarrow 53.3 - 24.5 = 0.9x$

$\Rightarrow 28.8 = 0.9x$

$\Rightarrow x = \frac{28.8}{0.9} = 32$ cm

$\therefore$ When y = 53.3 inches, x = 32 cm

@@@ Thank you @@@